Convergence onto a Non-Fixed-Point by Tracking:

A Decoupled Repertoire–Selector Architecture for Responsive Stability

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Abstract

A companion paper argued that a single fixed-point operator unifies computation across domains, but observed a sharp limit: a fixed point describes stable *being*, whereas many systems of interest—perception, control, cognition, life—exhibit stable *becoming*, perpetual structured motion that never settles. This paper studies a clean mechanism for the latter. We show that the apparent opposition between stability and non-settling dissolves once two functions that are usually fused are *decoupled*: the *repertoire* (the set of available states) and the *selector* (the mechanism that designates which state is active). When these are separated, a system can be *always converged*—instantaneously fitted to current demand—while *never settling*, because the demand it tracks keeps moving. We call this convergence onto a non-fixed-point by tracking. We instantiate it concretely as a routing structure whose repertoire is a geometrically grounded set of seven ports (three interior and four boundary, arising from an inscribed triangle and a circumscribed square sharing one circle), and whose selector is a continuous, demand-driven fit. A hard architectural constraint—emission out of the system occurs only through boundary ports—is enforced by the geometry and verified to hold exactly across random demand (0 violations in 5000 trials). Driven by a demand stream that explores its input space, the router produces a port sequence that is *structured* (drawn only from the seven ports) yet *non-repeating* (92 distinct length-three subsequences; bigram entropy 4.62 bits; each port leading to four or five distinct successors), the formal signature of lawful becoming rather than either a fixed cycle or chaos. We are explicit about scope: the selector is, in conventional terms, a continuous attention / mixture-of-experts mechanism, and the structure of the output is supplied by the demand stream rather than generated intrinsically; the contribution is the decoupling principle, the geometric grounding, the enforced two-tier constraint, and the interpretation as a resolution of the being/becoming tension. We close by locating the genuine frontier: the architecture *tracks* a fixed repertoire, whereas open-ended development would require the repertoire and the selector themselves to be reshaped by the history of what flows through them.

1 Introduction

A fixed point $x^\star = S(x^\star)$ is the mathematical form of stable being: apply the dynamics and nothing moves. A companion paper [1] argued that a single fixed-point operator, applied to suitably grounded representations, unifies computation across domains as varied as electronic structure, associative memory, and constraint satisfaction. That work also identified a sharp boundary. Many of the systems we most want to model are not at rest: a controller tracking a moving target, an organism maintaining itself far from thermodynamic equilibrium, an attentional system shifting among foci, a mind moving among states. These exhibit *stable becoming*—bounded, structured, perpetual motion that never settles to a point. A theory of fixed points appears, at first, to be exactly the wrong tool for them.

This paper shows that the opposition is only apparent, and that it dissolves under a single architectural move: *decoupling the repertoire from the selector*. By the repertoire we mean the set of states a system can occupy; by the selector we mean whatever designates which state is currently active. In many models these are fused into one object—a single trajectory that must simultaneously *be* the states and *choose* among them. We show that this fusion is the source of a dilemma (a single trajectory over a small symmetric state set tends to collapse either to a fixed point or to a rigid cycle), and that separating the two functions removes it. Once separated, the selector can be *continuous* and *demand-driven*: at every instant it converges to the state that best fits the current demand, and because the demand moves, the active state moves with it. The system is therefore *always converged* and *never settled* at the same time. We call this **convergence onto a non-fixed-point by tracking**: the invariant object is not a resting state but the ongoing, fitted correspondence between demand and selection.

We make the idea concrete with a specific, geometrically grounded instance. The repertoire is a set of seven ports obtained from a classical "squaring of the circle" figure: a circle, a triangle inscribed in it (three vertices), and a square circumscribed about it (four tangency points). Because the three-fold and four-fold symmetries overlay on one circle and $\gcd(3, 4) = 1$, the seven ports occupy distinct angular positions and are therefore non-degenerate—seven genuinely different transfer functions rather than seven copies of one. The selector is the circle itself, read as a continuous router that, driven by demand, emits through whichever port best fits. A two-tier constraint distinguishes the tiers: the four boundary (square) ports are the only ones through which output crosses out of the system, while the three interior (triangle) ports carry internal transfer only.

Our contributions are:

1. **A principle.** We articulate repertoire–selector decoupling as the mechanism that converts a fixed-point operator into a tracker, and we state precisely the sense in which the resulting system is always converged yet never settles (Section 3).
2. **A grounded instance.** We construct the squared-circle router: a seven-port manifold with a provably non-degenerate angular structure and a continuous demand-driven selector with a mixed geometric/functional fit (Section 4).
3. **An enforced two-tier constraint.** We show that emission out of the system can be restricted to boundary ports by the geometry itself, and verify the constraint holds exactly under random demand (Section 5).
4. **Lawful becoming, demonstrated and bounded.** We show that demand which explores the input space yields a structured but non-repeating port sequence, and we are explicit that this structure is transduced from the demand rather than generated intrinsically (Sections 5–7).

We state plainly what we do not claim. The selector is, in standard terms, a continuous attention or mixture-of-experts gate; we introduce no new gating mathematics. The richness of the output sequence is inherited from the demand stream. And, as we argue at the close, tracking a *fixed* repertoire is not yet open-ended development. The value of the work is the decoupling principle, the specific geometric grounding, the enforced constraint, and the interpretation that resolves a stated tension between stability and becoming.

2 Related Work

The companion fixed-point framework. This paper is the sister of [1], which develops the fixed-point operator and demonstrates its cross-domain reach; we adopt its vocabulary (driver, cut, grounding) and address the limitation it identifies.

Repertoire/selector separation. The separation of a stored repertoire from a context-driven selector recurs across computing and biology: the stored-program (memory/processor) distinction [2]; the genotype/phenotype distinction in which a fixed genome is selectively expressed by context; and, in machine learning, content-based attention [3, 4] and mixture-of-experts gating [5, 6], in which a continuous gate routes computation to a subset of a fixed expert set. Our selector is, mathematically, a gate of this family; our emphasis is on the geometric grounding of the repertoire and on the interpretation as responsive stability.

Tracking and moving-target convergence. Convergence to a moving reference is the defining concern of control and estimation: tracking controllers, observers, and recursive filters maintain an estimate that is always "converged" to a target that itself evolves [7]. We borrow this stance and apply it to state selection: the selected state is a tracked quantity, perpetually re-fitted.

Structured non-settling dynamics. Bounded, recurrent, non-settling behavior is studied as limit cycles, quasiperiodicity, heteroclinic cycles [8], and strange attractors; in particular, attempts to generate aperiodic itineraries among saddles (heteroclinic networks) are known to require delicate parameter tuning [9]. Part of our motivation is that a decoupled selector achieves structured non-repetition *without* this knife-edge tuning, by transducing demand rather than relying on intrinsic instability.

Geometric grounding. The squaring-of-the-circle figure is classical; our use of it is to obtain a port set whose angular signatures are non-degenerate by the coprimality of the inscribed and circumscribed symmetries. This is a design choice that makes the repertoire's elements distinguishable, not a claim about the figure's traditional significance.

3 The Principle: Decoupling Converts a Fixed Point into a Tracker

3.1 The fusion dilemma

Consider modelling a system with a small number of qualitatively distinct states as a single trajectory in a state space, where the states are attractors and transitions are the trajectory's motion between them. Two failure modes are generic. If the attractors are strong, the trajectory settles into one: a fixed point, no becoming. If a global circulation is added to prevent settling, the trajectory traverses the states in a fixed order: a rigid cycle, structured but not responsive. Achieving a *structured but non-repeating* itinerary from such a single trajectory requires placing the system at the boundary between order and chaos (e.g. a robust heteroclinic network), which is known to demand delicate tuning [9, 8]. The difficulty is intrinsic to the *fusion*: one object is being asked both to hold the repertoire and to select within it.

3.2 Decoupling and tracking

Separate the two functions. Let a fixed *repertoire* be a set of ports $\{q_i\}_{i=1}^N$, each a fixed transfer function with a signature s_i. Let a *selector* be a map from a time-varying *demand* $u(t)$ to an emission distribution $w(t) \in \Delta^{N-1}$ over the ports, defined by a fit between the demand and each port's signature:

$$w_i(t) = \frac{\exp\bigl(\beta\, \phi(u(t), s_i)\bigr)}{\sum_j \exp\bigl(\beta\, \phi(u(t), s_j)\bigr)},$$

with ϕ a fit function and β a sharpness. At each instant the selector is *converged*: $w(t)$ is the unique maximizer of expected fit at sharpness β, i.e. a fixed point of the instantaneous selection problem. Yet the selected port $\arg\max_i w_i(t)$ *moves* as $u(t)$ moves. The system is therefore always converged and never settled.

Definition 1 (Convergence by tracking)**.** *A system exhibits* convergence onto a non-fixed-point by tracking *if, for every t, its state is the (converged) solution of an instantaneous fit problem against a target $u(t)$, while $u(t)$ is non-stationary so that no time-independent state is a fixed point of the closed system. The invariant object is the correspondence $u \mapsto \arg\max_i \phi(u, s_i)$, not any state.*

This reframes the being/becoming opposition. Stability is no longer "at rest forever" but "instantaneously fitted"; becoming is no longer "unstable" but "the fitted state tracking a moving world." The fixed-point operator of [1] is not discarded; it is relocated from the *state* to the *selection*, and re-solved at every instant against a moving demand. Crucially, because the repertoire is held by a separate, static structure, the selector is free to be continuous and demand-driven, and the fusion dilemma does not arise: a non-repeating output requires only a non-periodic demand, not a knife-edge dynamical regime.

4 Architecture: The Squared-Circle Router

4.1 The repertoire: a non-degenerate seven-port manifold

We ground the repertoire in the squaring-of-the-circle figure. On a unit circle, place an inscribed equilateral triangle and a circumscribed square. The triangle contributes three *interior* ports at its vertices; the square contributes four *boundary* ports at its tangency points with the circle. The port angles (in degrees) are

$$\underbrace{90,\ 210,\ 330}_{\text{interior (triangle)}} \qquad \underbrace{0,\ 90,\ 180,\ 270}_{\text{boundary (square)}}.$$

Proposition 1 (Non-degeneracy)**.** *Overlaying a k_1-fold and a k_2-fold regular arrangement on a common circle yields $k_1 + k_2$ angular positions with no nontrivial rotational symmetry relating the two sets when $\gcd(k_1, k_2) = 1$. For $(k_1, k_2) = (3, 4)$ the seven ports occupy positions that are not interchangeable under a single symmetry, so they constitute seven distinct transfer functions.*

The overlay places one interior vertex and one boundary tangency on a shared axis (both at 90° in the canonical placement); we treat this as a design parameter (a deliberately "doubled" axis) that may be removed by rotating the triangle. Figure 1(a) shows the figure and the seven ports.

4.2 The selector: a continuous demand-driven router with mixed fit

Demand is a pair $u = (\hat{d}, \eta)$: a unit *direction* $\hat{d}$ in task space and a scalar *externality* $\eta \in [0,1]$ measuring how strongly the task must cross out of the system. The fit of demand to port i (with unit direction v_i on the circle and tier indicator $b_i \in \{0,1\}$, $b_i = 1$ for boundary) is a mixture of a geometric and a functional term:

$$\phi(u, s_i) = \underbrace{w_g\,(v_i \cdot \hat{d})}_{\text{geometric alignment}} + \underbrace{w_f\,\big(1 - |b_i - \eta|\big)}_{\text{functional / tier match}} + \log\big(g_i(\eta) + \epsilon\big),$$

where the geometric term rewards directional alignment, the functional term rewards tier match (boundary ports match high externality, interior ports match low), and $g_i(\eta)$ is the two-tier gate below. The emission distribution is the softmax of ϕ at sharpness β. The mixture is intentional: routing follows *both* where the demand points and *what kind* of task it is.

4.3 The two-tier constraint: emission out only through the boundary

We enforce that output crosses out of the system only through boundary ports. The gate

$$g_i(\eta) = \begin{cases} 1 & b_i = 1 \text{ (boundary)} \\ 1 - \eta & b_i = 0 \text{ (interior)} \end{cases}$$

appears as $\log g_i$ in the score, so that as externality $\eta \to 1$ the interior ports are driven to zero emission probability, while boundary ports remain available; for internal demand ($\eta \to 0$) all ports are open and the geometric term dominates. Thus interior (triangle) ports carry internal transfer and can never emit out, while boundary (square) ports are the sole egress. This realizes, as a structural property of the geometry, the requirement that the interior route only within and the boundary routes out.

5 Results

5.1 Geometry and continuous handoff

Figure 1(a) shows the seven-port manifold; the sorted port angles have distinct gaps, confirming non-degeneracy. Figure 1(b) shows the emission distribution over the four boundary ports as the demand direction sweeps a full circle at high externality: each boundary port owns a contiguous angular sector with smooth crossovers, so the router hands off continuously from one port to the next rather than switching discretely. Near a sector boundary the emission is shared between two adjacent ports (the continuous "blend" regime), realizing the intended continuity of the selector.

5.2 Two-tier gating

Figure 2 shows the routed probability mass on the boundary tier and the interior tier as a function of demand externality, averaged over demand directions. At low externality the interior tier carries essentially all routing (internal transfer); as externality increases the mass transfers to the boundary tier (emission out), crossing over near $\eta = 0.5$. The constraint is exact, not merely statistical: across 5000 random demands (random direction and externality), the number of out-emissions through interior ports was **0**. Emission out of the system occurs only through boundary ports.

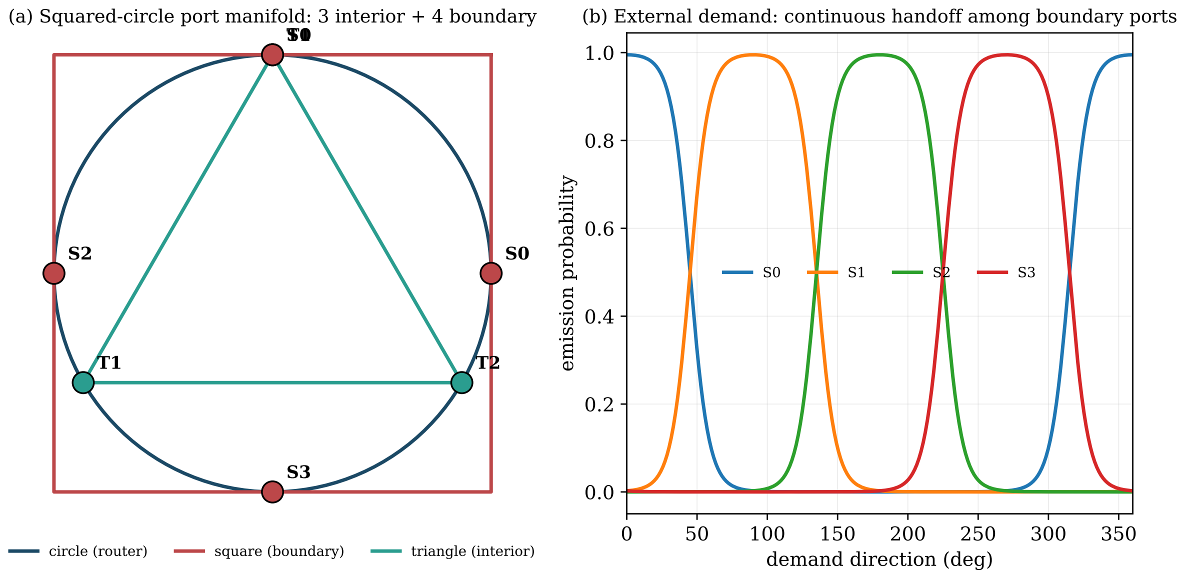

Figure 1: (a) The squared-circle port manifold: a circle (router), an inscribed triangle (three interior ports, teal) and a circumscribed square (four boundary ports, red). The 3-fold and 4-fold overlays are coprime, so the seven ports occupy distinct angles. (b) With high-externality demand, emission passes continuously among the four boundary ports as the demand direction rotates; each owns a sector with smooth handoffs.

5.3 Tracking: structured but non-repeating selection

We drive the router with a demand stream whose direction sweeps the full circle (with jitter) and whose externality oscillates across $[0, 1]$, so that the demand explores the input space. The resulting port sequence (Figure 3) has the signature of lawful becoming:

- **All seven ports are used**, confirming the full repertoire is reachable.
- **It is non-settling**: over the run the active port is re-selected perpetually; at every instant there is a definite dominant port (always converged) yet the dominant port keeps changing (never settled), Figure 3(a).
- **It is structured but non-repeating**: the distinct-port itinerary (length 692) contains **92** distinct length-three subsequences out of 690—far from the ≤ 4 that a fixed cycle would yield—with a bigram entropy of **4.62** bits. Each port leads to four or five distinct successors (Figure 3(b)), so the selection graph is richly branching rather than a single loop.
- **Both tiers are exercised**: about 65% of the time the router emits out (boundary ports) and 35% routes internally (interior ports), tracking the externality of the demand.

This is precisely the structured-non-repeating behavior that a single fused trajectory could not achieve without knife-edge tuning (Section 3); here it follows directly from a non-periodic demand applied to a decoupled selector.

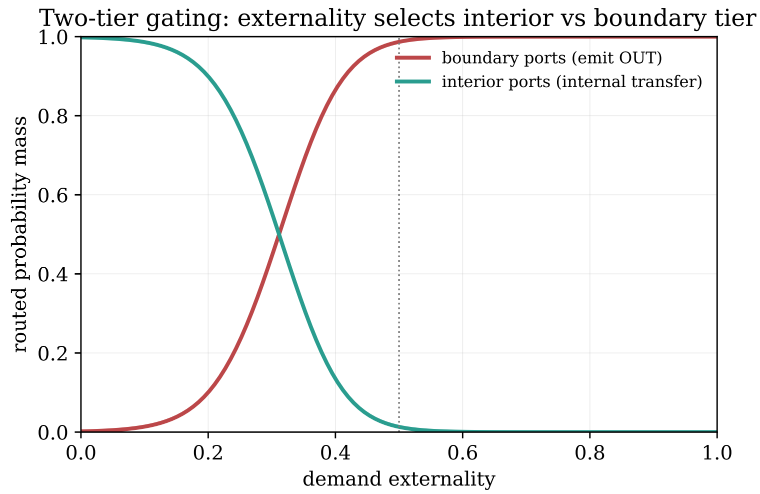

Figure 2: Two-tier gating. Routed probability mass on interior (teal) versus boundary (red) ports as demand externality varies, averaged over demand directions. Internal demand routes to interior ports; external demand is forced to boundary ports. Out-emission through interior ports never occurs (0/5000 random demands).

6 Discussion

6.1 What the architecture resolves

The companion framework [1] captured stable being but not stable becoming. The decoupling principle resolves this without abandoning the fixed-point operator: it relocates the fixed point from the state to the instantaneous selection, and tracks a moving demand. The squared-circle router shows the resolution concretely. It is bounded (the square encloses the manifold), continuous (the selector hands off smoothly), and perpetually non-settling (it tracks demand), and it produces structured but non-repeating output. The key conceptual yield is that *stability and becoming are not opposites once selection is decoupled from storage*: the system is stable in the sense of being always fitted, and becoming in the sense that the fit tracks a changing world.

6.2 Why decoupling, specifically, is the lever

The fusion dilemma (Section 3) shows that a single object holding both repertoire and selector is forced toward either rest or rigid cycling, and that escaping into structured non-repetition requires a delicate dynamical regime [9]. Decoupling removes the constraint at its source: with a static repertoire, the selector need not generate stability dynamically (the ports are stable by construction), and so it is free to be a smooth, demand-driven gate whose output inherits whatever structure the demand carries. The separation also makes the two-tier constraint natural: tiers are a property of the static repertoire (interior vs. boundary ports), enforced once, rather than something the dynamics must continually maintain.

6.3 Honest placement

The selector is, in conventional terms, a continuous content-based gate of the attention / mixture-of-experts family [3, 4, 5, 6]; we contribute no new gating mathematics. The geometric grounding

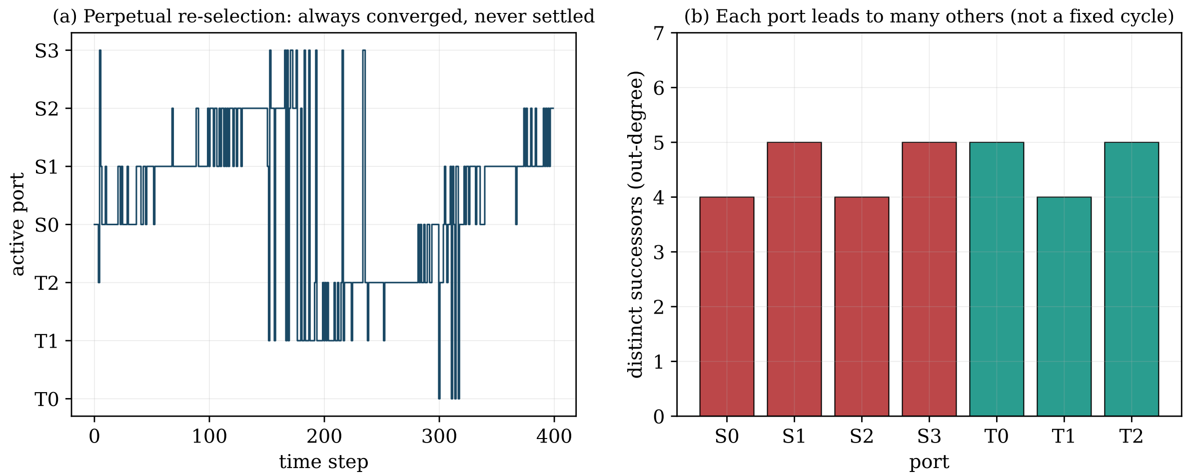

Figure 3: Tracking produces lawful becoming. (a) Active port over time (first 400 steps): the selection is re-fitted at every step (always converged) while the active port changes perpetually (never settled). (b) Out-degree per port over the full run: each port is followed by four or five distinct successors, so the itinerary is a richly branching, non-repeating sequence (92 distinct length-3 words; bigram entropy 4.62 bits), not a fixed cycle. Teal: interior ports; red: boundary ports.

(a non-degenerate 3+4 port set from the squared-circle figure) and the enforced two-tier egress constraint are specific design choices, and the framing as a resolution of the being/becoming tension is interpretive. As in the companion paper, the novelty is a conjunction and a perspective rather than a new kernel: the decoupling principle, the grounded repertoire, the enforced constraint, and the tracking interpretation, assembled into one working architecture.

6.4 The genuine frontier: tracking versus development

The most important limitation is also the clearest research direction. The router *tracks* a *fixed* repertoire: the ports, their signatures, and the fit function are all static; only the demand moves. This yields *responsive* stability—a system that adapts its output to a changing world—but not *development*: the repertoire does not grow, and the selector does not rewrite what counts as a good fit. A system that is alive in the strong sense does both—it acquires new states and revises its own criteria in light of its history. Formally, this is the step from a static $(\{s_i\}, \phi)$ to a pair that is itself updated by the stream of demands and emissions, $\{s_i\}_t$ and ϕ_t, so that the repertoire and the selector co-evolve with use. That step—from tracking to development, from a fixed manifold of ports to one reshaped by what flows through it—is, we believe, the substantive next problem, and the present architecture sharpens exactly where it lies.

7 Limitations

- **The selector is a known mechanism.** Continuous demand-based routing over a fixed set of experts is standard [4, 6]; the contribution is the decoupling principle, the geometric grounding, the enforced constraint, and the interpretation, not the gate.
- **Output structure is transduced, not generated.** The non-repeating itinerary reflects the structure of the *demand* stream. With periodic demand the output is periodic. The

router is a faithful transducer of demand into structured emission, not an autonomous source of novelty.

- **Fixed repertoire.** The ports, signatures, and fit function are static; the architecture tracks but does not develop (Section 6).
- **Geometric grounding is a design choice.** The squared-circle figure is used to obtain non-degenerate port signatures; the construction does not depend on, or assert, any traditional meaning of the figure. The 90° interior/boundary coincidence is a parameter, removable by rotation.
- **Demand modelling.** We model demand as a direction plus a scalar externality. Real applications would require a principled mapping from task state to this demand, which we do not address here.
- **Still a fixed-point computation underneath.** Tracking re-solves an instantaneous fit (a fixed point of selection) at each step; it relocates, but does not eliminate, the fixed-point character identified in [1].

8 Conclusion

A fixed point describes stable being; many systems require stable becoming. We showed that the opposition dissolves once the repertoire (available states) and the selector (which state is active) are decoupled: the resulting system is always converged—instantaneously fitted to current demand—yet never settles, because the demand it tracks keeps moving. We made this concrete as the squared-circle router, whose repertoire is a non-degenerate seven-port manifold (three interior, four boundary) grounded in the squaring-of-the-circle figure, and whose selector is a continuous, demand-driven, mixed-fit gate with a two-tier egress constraint enforced by the geometry and verified exact (0/5000 violations). Driven by demand that explores its input, the router produces structured but non-repeating selection (92 distinct length-three words; 4.62 bits), the signature of lawful becoming, without the knife-edge tuning that an equivalent fused dynamical system would require. The contribution is the decoupling principle, the grounded repertoire, the enforced constraint, and the interpretation that reconciles stability with becoming. The frontier it sharpens is the move from tracking a fixed repertoire to developing one—letting the ports and the fit criterion be reshaped by the history of what flows through them—which is where a responsive system would become a developing one.

Acknowledgements

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Reproducibility

The router and the dynamic driver are small, self-contained NumPy programs; all figures and reported statistics (port angles, the 0/5000 constraint check, the 692-length itinerary with 92 distinct length-three subsequences and 4.62-bit bigram entropy, and the per-port out-degrees) are generated directly from the experiment script.

References

[1] B. D. Horn, "A substrate-independent fixed-point operator for composable computation: one convergence driver across electronic structure, associative memory, and constraint satisfaction," Prometheus7 Research Institute (companion paper).

[2] J. von Neumann, "First draft of a report on the EDVAC," Moore School of Electrical Engineering, University of Pennsylvania (1945); reprinted in *IEEE Ann. Hist. Comput.* **15**, 27–75 (1993).

[3] D. Bahdanau, K. Cho, and Y. Bengio, "Neural machine translation by jointly learning to align and translate," *ICLR* (2015).

[4] A. Vaswani *et al.*, "Attention is all you need," *Adv. Neural Inf. Process. Syst.* **30** (2017).

[5] R. A. Jacobs, M. I. Jordan, S. J. Nowlan, and G. E. Hinton, "Adaptive mixtures of local experts," *Neural Comput.* **3**, 79–87 (1991).

[6] N. Shazeer *et al.*, "Outrageously large neural networks: the sparsely-gated mixture-of-experts layer," *ICLR* (2017).

[7] R. E. Kalman, "A new approach to linear filtering and prediction problems," *J. Basic Eng.* **82**, 35–45 (1960).

[8] J. Guckenheimer and P. Holmes, "Structurally stable heteroclinic cycles," *Math. Proc. Camb. Philos. Soc.* **103**, 189–192 (1988).

[9] P. Ashwin and M. Timme, "Unstable attractors: existence and robustness in networks of oscillators with delayed pulse coupling," *Nonlinearity* **18**, 2035–2060 (2005).